The 180° Hybrid Coupler

The 180° **Hybrid Coupler** (sometimes know as the "ring" or "rat-race" hybrid) is a lossless, matched and reciprocal 4-port device, with a scattering matrix of the **anti-symmetric** form:

$$\bar{\bar{S}} = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

Just like the quadrature coupler, however, we find that:

$$\alpha = \beta = \frac{1}{\sqrt{2}}$$

So that the scattering matrix for this device is:

$$\bar{\bar{S}} = \begin{bmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 0 & 0 & -1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 0 & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}$$

Hence, this coupler is likewise a **3dB coupler**—the power into a given port (with all other ports matched) is equally divided between two of the three output ports.

Note the relative phase between the outputs, however, is **dependent** on which port is the input.

For example, if the **input** is port 1 or port 3, the two signals will be **in phase**—no difference in their relative phase!

However, if the input is port 2 or port 4, the output signals will be 180° **out of phase** ($e^{j\pi} = -1$)!

An interesting application of this coupler can be seen if we place **two input signals** into the device, at ports 2 and 3. Note the signal out of port 1 would therefore be:

$$V_1^-(z) = S_{12}\,V_2^+(z) + S_{13}\,V_3^+(z)$$
$$= \frac{1}{\sqrt{2}}\left(V_3^+(\ ) + V_2^+(\ )\right)$$

while the signal out of port 4 is:

$$V_4^-(z) = S_{42}\,V_2^+(z) + S_{43}\,V_3^+(z)$$
$$= \frac{1}{\sqrt{2}}\left(V_3^+(z) - V_2^+(z)\right)$$

Note that the output of port 1 is proportional to the **sum** of the two inputs. Port 1 of a 180° Hybrid Coupler is thus often referred to as the **sum** (Σ) port.

Likewise, port 4 is proportional to the **difference** between the two inputs. Port 4 a 180° Hybrid Coupler is thus often referred to as the **delta** (Δ) port.

There are **many** applications where we wish to take the sum and/or difference between two signals!

The 180° Hybrid Coupler can likewise be used in the **opposite** manner. If we have **both** the sum and difference of two signals available, we can use this device to separate the signals into their separate components!